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SU(3) surface tension from the lattice with the fixed point action[1]

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Abstract

The surface tension at the deconfinement transition of SU(3) is determined with a parametrized version of the fixed point action of a renormalization group transformation on lattices with temporal extent $N_\tau = 3$ and 4 and spatial extent $N_\sigma/N_\tau = 3$ and 4. A considerable cut-off dependence can be seen in comparison with earlier determinations from tree level Symanzik and tadpole improved actions.

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1 Introduction

Lattice Monte Carlo simulations are at present the most effective tool to study SU(3) at finite temperature both in the gluon plasma phase, where perturbation theory is plagued by the well known infrared problem [1], and at the critical temperature T_c, i.e. in a highly non-perturbative regime. Lattice calculations of the bulk thermodynamic quantities in the deconfined phase of SU(3) with the Wilson action are affected, however, by a large cut-off dependence [2], which is evident also in perturbation theory in the high temperature ideal gas limit [3]. The continuum limit can be extrapolated with the Wilson action only from lattices with temporal extent N_τ not smaller than 6 [2].[2] Recent calculations [3, 4] have revealed that such cut-off dependence can be drastically suppressed, from temperatures slightly above T_c onwards, if tree level Symanzik [5] or tadpole [6] improved actions are used in numerical simulations. In particular, with the tadpole improved action the free energy density shows no scaling violation from $N_\tau = 3$ to $N_\tau = 4$ and is consistent with the continuum extrapolated from the Wilson action results. Moreover, the lattice determinations of the energy density minus three times the pressure from the tree level Symanzik improved action at $N_\tau = 4$ are in agreement with the continuum. The latter result suggests that the cut-off effects beyond the tree level order are relatively unimportant in the high temperature phase of SU(3) [4]. In other words, the tree level improvement alone can eliminate most of the cut-off dependence.

Fixed point actions [7] are lattice actions living in the space of couplings on the straight line which originates at the fixed point (FP) of a renormalization group (RG) transformation and leaves the critical surface along the direction of the coupling g^2. They are *classical* perfect actions, i.e. their spectral properties are free of cut-off effects at the classical level. For this reason and in view of the previous considerations, FP actions are expected to yield a remarkable improvement in lattice studies of SU(3) thermodynamics.

Given an arbitrary RG transformation, the FP action of any lattice configuration is well defined and can be determined numerically by multigrid minimization [7, 8]. For practical reasons, however, in Monte Carlo simulations only simple enough *parametrizations* of the FP action can be used. Being a potential source of cut-off effects, the parametrization of a FP action is a very delicate task. In SU(3) lattice gauge theory, the parametrizations proposed so far are of the form

$$S = \beta \, \frac{1}{N} \sum_{C} \sum_{i \geq 1} c_i(C) [N - \mathrm{Re}\,\mathrm{Tr}(U_C)]^i \quad , \qquad \beta \equiv \frac{2N}{g^2} \quad , \tag{1}$$

[2] In lattice simulations the temperature T and the volume V are determined by the lattice size $N_\sigma^3 \times N_\tau$, $N_\tau < N_\sigma$, through $T = 1/(N_\tau a)$ and $V = (N_\sigma a)^3$.

where the first sum is over all the closed paths C and U_C is the product of the link variables $U_\mu(n)$ along the path C. The couplings $c_i(C)$ are determined by a fit procedure on the numerically estimated "exact" values of the FP action of a representative set of Monte Carlo configurations [9, 10]. In Ref. [10] the arbitrariness in the choice of the RG transformation has been exploited to define a very rotationally symmetric RG transformation, optimized in order that the related FP action is very short-ranged and, therefore, easier to be parametrized. This RG transformation has been called for historical reasons "type III" RG transformation. The related FP action has been parametrized with the plaquette and the twisted perimeter-6 loop (see Fig. 1), with the couplings $c_i(C)$, $i = 1, \ldots, 4$, given in Table 1.

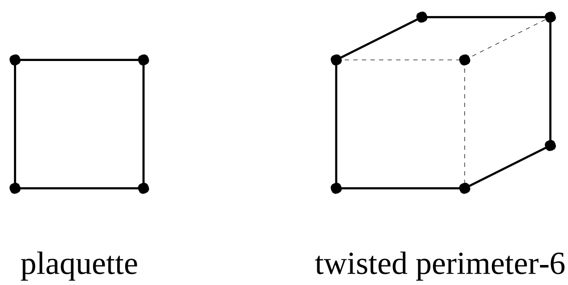

Figure 1: Loops considered in the type III parametrized FP action.

In Ref. [11] the type III parametrized FP action has been used to determine from the lattice the free energy density of SU(3) at $T/T_c = 4/3,\ 3/2,\ 2$. Simulations were performed on lattices as small as $8^3 \times 2$ and $12^3 \times 3$. Results at $N_\tau = 3$ are in agreement with the continuum as extrapolated with the Wilson action from lattices with $N_\tau = 6$ and 8 [2] and with the results from the tadpole improved action at $N_\tau = 3$ and 4 [4] (for a complete review, see also [12]).

Table 1: Couplings of the type III parametrized FP action [10].

	c_1	c_2	c_3	c_4
plaquette	0.4822	0.2288	–0.1248	0.0228
twisted perimeter-6	0.0647	–0.0224	0.0030	0.0035

In this paper, the same parametrized FP action has been used to determine the surface tension of SU(3) on lattices with temporal extent $N_\tau = 3$ and 4 and spatial extent $N_\sigma/N_\tau = 3$ and 4. Like the latent heat, the surface tension is a physical quantity characteristic of the first order deconfinement transition of SU(3).[3] As a consequence, it is affected by the properties of both the low and the high temperature phase. Since in the latter phase the use of the type

[3]If the high temperature phase transition of the full QCD were first order, the surface

III parametrized FP action allows an impressive reduction of the cut-off dependence of the thermodynamic quantities, it is interesting to check if the same occurs for a quantity typical of the critical region.
The results for the surface tension obtained in this work have been then compared with earlier determinations from the Wilson action [13], tree level Symanzik and tadpole improved actions [14].

2 Monte Carlo results for the surface tension

At the critical temperature of the first order deconfinement transition of SU(3), there can be mixed states where the confined and the deconfined phases coexist, separated by an interface. These mixed states have an additional free energy $F = \sigma A$ (σ is the surface tension, A the area of the interface), which indicates that they are less probable than pure states of one or the other phase. The frequency distribution of any order parameter Ω at the transition has a typical double-peak structure, where the two peaks correspond to the pure phase configurations, while the region in-between corresponds to configurations containing an interface. The peaks become more pronounced when the volume is increased. The probability distribution of Ω is given by [13]

$$\begin{aligned} P(\Omega) &= c_1 \mathrm{e}^{-f_1 V/T} \mathrm{e}^{-(\Omega-\Omega_1)^2/d_1^2} + c_2 \mathrm{e}^{-f_2 V/T} \mathrm{e}^{-(\Omega-\Omega_2)^2/d_2^2} \\ &+ c_m \mathrm{e}^{-(f_1 V_1 + f_2 V_2 + 2\sigma A)/T} \quad , \\ & \quad c_i \propto V^{1/2}, \quad d_i \propto V^{-1/2}, \qquad i = 1,2 \quad , \end{aligned} \tag{2}$$

where f_1, f_2 are the free energy densities of the two pure phases and V_1, V_2 are their volumes. The factor 2 in the free energy of the interface appears since at finite volume with periodic boundary conditions two interfaces are needed to separate two volumes. In the previous formula, the dependence on Ω enters through the relation $\Omega = (V_1\Omega_1 + V_2\Omega_2)/V$. At T_c and at infinite volume, the free energy densities in both phases are identical, $f_1 = f_2$. The leading volume dependence of the surface tension is determined by [13]

$$\left(\frac{\sigma}{T_c^3}\right)_V = -\frac{1}{2}\left(\frac{N_\tau}{N_\sigma}\right)^2 \ln\left(\frac{P_{\mathrm{min}}}{P_{\mathrm{max},1}^{\gamma_1} P_{\mathrm{max},2}^{\gamma_2}}\right) \quad , \tag{3}$$

where P_{min} is the minimum of the distribution $P(\Omega)$, $P_{\mathrm{max},1}$ and $P_{\mathrm{max},2}$ are the two maxima corresponding to the values Ω_1 and Ω_2 of Ω in the pure states of the two phases at infinite volume. The weights γ_1 and γ_2 are determined imposing $\langle\Omega\rangle = \gamma_1\Omega_1 + \gamma_2\Omega_2$, with $\gamma_1 + \gamma_2 = 1$.

tension would play an important role in the quark-gluon plasma formation in heavy-ion experiments and in the nucleation of hadronic matter in the early universe [13].

Table 2: Parameters of the runs and results for β_c and χ_L/N_σ^3. Errors have been estimated by the jackknife method.

lattice	# β	# iterations	β reweighting	β_c	χ_L/N_σ^3
$12^3 \times 3$	3	645099	3.58995	3.58982(9)	$2.958(20) \times 10^{-2}$
$12^3 \times 4$	1	171209	3.69915	3.7007(3)	$1.013(16) \times 10^{-2}$
$16^3 \times 4$	3	135662	3.70025	3.7009(5)	$8.87(20) \times 10^{-3}$

A convenient choice for the order parameter in SU(3) is the absolute value of the Polyakov loop $L = 1/N_\sigma^3 \sum_{\vec{n}} \mathrm{Tr} \prod_{n_4=1}^{N_\tau} U_\mu(\vec{n}, n_4)$. Numerical simulations have been performed on three lattices at values of β close to criticality. A summary of the parameters of the runs is given in Table 2. Here one iteration means the combination of a 20-hit Metropolis and 4 over-relaxation updatings; the runs have been done on DEC-alpha machines. In all the numerical simulations frequent flips of the order parameter $|L|$ could be observed between the two phases, thus indicating a proper sampling of the thermal ensemble at criticality. The occurrence of frequent jumps was expected in view of the relatively low N_σ/N_τ ratio. The critical couplings have been determined through the location of the peak in the Polyakov loop susceptibility $\chi_L = N_\sigma^3(\langle |L|^2\rangle - \langle |L|\rangle^2)$. They are in agreement within errors with the results of Ref. [10] where the so called Columbia definition was adopted.[4] The Polyakov loop distribution on the $12^3 \times 3$ lattice and on the two lattices with $N_\tau = 4$ is presented in Fig. 2. Data at different β on the same lattice have been interpolated by Ferrenberg-Swendsen reweighting in order to make the peaks in the Polyakov loop distribution have the same height. In order to determine $P_{\min}$, $P_{\max,1}$ and $P_{\max,2}$, the statistical fluctuations in the vicinity of the extrema of $P(|L|)$ have been smoothed out by a polynomial fit.

In Table 3 the results of this work for σ/T_c^3 are compared with other determinations from the Wilson action [13], the tree level Symanzik improved action (plaquette + (1×2) loop) and the tadpole improved action (with the same loops) [14]. The infinite volume extrapolations have been done according to the ansatz [13]

$$\left(\frac{\sigma}{T_c^3}\right)_V = \left(\frac{\sigma}{T_c^3}\right) - \left(\frac{N_\tau}{N_\sigma}\right)^2 \left[c + \frac{1}{4}\ln N_\sigma\right] . \tag{4}$$

The result on the $12^3 \times 3$ lattice and the infinite volume extrapolation at $N_\tau = 4$ for the tadpole improved action indicate no cut-off dependence and a continuum value of σ/T_c^3 equal to 0.0155(16) (corresponding to $\sigma \sim 7$ MeV/fm^2).[5]

[4] This comparison is important since in Ref. [11] the physical scale in the determination of the free energy density was set using the critical couplings as obtained in Ref. [10].

[5] Here and in the following it is assumed that the infinite volume extrapolation at $N_\tau = 3$ does not differ too much from the result on the $12^3 \times 3$ lattice.

Table 3: σ/T_c^3 for various lattice actions on several lattices. Column 7 gives the results for γ_1 from the type III parametrized FP action. Errors have been estimated by the jackknife method.

		Wilson [13]	tree level [14]	tadpole [14]	type III FP	
N_τ	volume	σ/T_c^3	σ/T_c^3	σ/T_c^3	σ/T_c^3	γ_1
3	12^3		0.0234(24)	0.0158(11)	0.0307(8)	0.420(10)
4	$12^2 \times 24$	0.0241(27)				
4	$24^2 \times 36$	0.0300(16)				
4	12^3				0.0196(11)	0.409(21)
4	16^3		0.0148(16)	0.0147(14)	0.0180(21)	0.441(23)
4	24^3		0.0136(25)	0.0119(21)		
4	32^3		0.0116(23)	0.0125(17)		
4	∞	0.0295(21)	0.0152(26)	0.0152(20)	0.026(5)	
6	20^3	0.0123(28)				
6	24^3	0.0143(22)				
6	$36^2 \times 48$	0.0164(26)				
6	∞	0.0218(33)				

Moreover, this continuum value is in agreement with the infinite volume extrapolation at $N_\tau = 4$ from the tree level Symanzik improved action and with the extrapolation in $1/N_\tau^2$ to $N_\tau = \infty$ of the results with the Wilson action at $N_\tau = 4$ and 6.
In the case of the type III parametrized FP action, the result on the $12^3 \times 3$ lattice and the infinite volume extrapolation at $N_\tau = 4$ are consistent with scaling, but at a value considerably larger than the continuum determined from the tadpole improved action. If one observes, however, that the infinite volume extrapolation at $N_\tau = 4$ is probably not reliable for the smallness of the lattices involved and assumes that the true extrapolated value cannot be too different from the result on the $16^3 \times 4$ lattice, then it must be concluded that there is a large scaling violation. Although the result on the $16^3 \times 4$ lattice is not inconsistent with the corresponding determinations from the tree level Symanzik and the tadpole actions, the result on the $12^3 \times 3$ lattice differs from the continuum more than in the case of the tree level Symanzik action.

This scenario would suggest that the type III parametrized FP action does not improve at $T = T_c$, at least as far as the surface tension is concerned, although it clearly does, from temperatures slightly above T_c onwards, in the case of the free energy density [11].

Before accepting this conclusion, possible sources of error have been investigated. The procedure followed to find the results of this work has been verified

by re-calculating some old results for the surface tension at $N_\tau = 2$ obtained with the Wilson action [15].

The use of the *naïve* Polyakov loop as order parameter, instead of the corresponding classically perfect operator, could represent in principle a source of lattice artifacts. This possibility has to be excluded, however, for several reasons: first of all, the type III parametrized FP action was optimized in Ref. [10] in order to dump the violation of rotational symmetry in the correlators of naïve Polyakov loops and, consequently, in the static $q\bar{q}$ potential. The tests performed in Ref. [10] provided impressively good results, in comparison with previous parametrizations of the FP action. Moreover, the surface tension depends only on the height of the minimum and of the maxima of the distribution $P(|L|)$, while possible lattice artifacts in the corresponding values of $|L|$ should play a minor role. This statement is supported by the fact that the nice results for the surface tension from the tadpole improved action of Ref. [14] were obtained with the naïve Polyakov loop. Finally, it has been explicitly checked in the present work that using the FP action density itself as order parameter instead of the absolute value of the Polyakov loop does not change the scenario, although the statistical fluctuations are larger.

The most natural explanation of the large observed cut-off dependence is that it is induced by the parametrization of the FP action, which perhaps fails to represent the exact FP action on configurations typical of the critical region. By comparing the results from the type III parametrized FP action with those of Ref. [14], one could also argue that planar (although less local) loops are preferable in lattice actions for simulations in presence of surfaces. The fact that the type III parametrized FP action does not satisfy the Symanzik condition at the tree level [16] could play a role in this respect.

It must be stressed, however, that the surface tension is a difficult quantity to determine for the high statistics required and for the complicated volume dependence. Further checks are necessary to confirm the previous conclusion.

3 Acknowledgments

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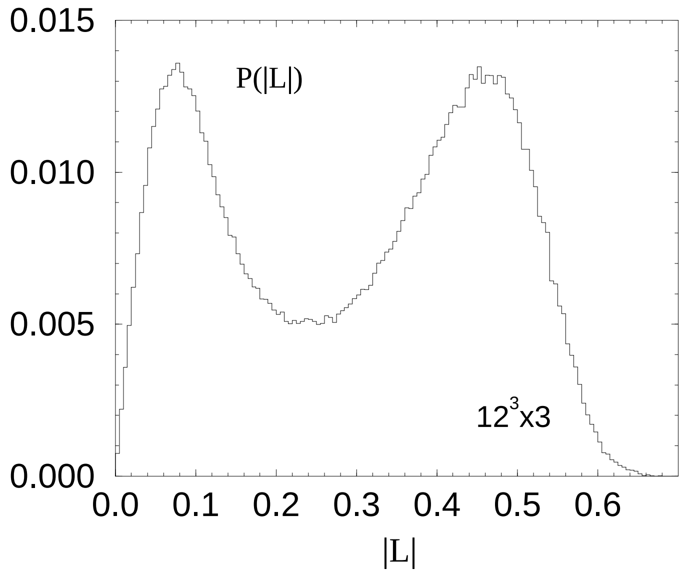

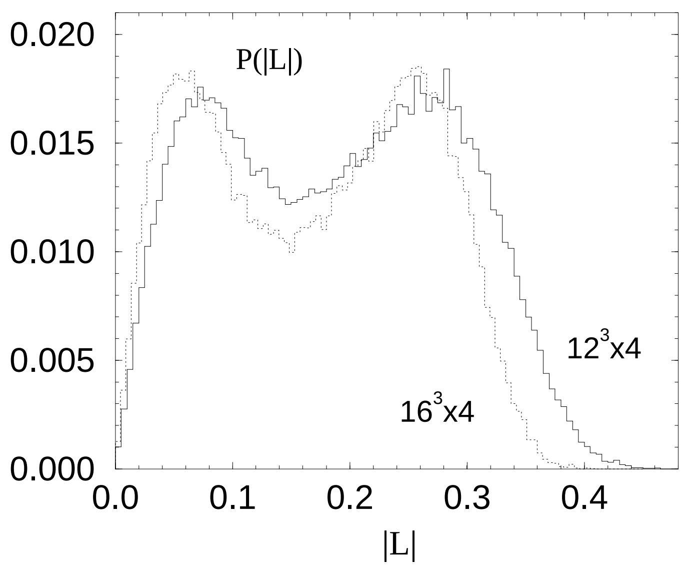

Figure 2: Polyakov loop distribution on the $12^3 \times 3$ lattice and on the $12^3 \times 4$ and $16^3 \times 4$ lattices. Data on the different lattices have been normalized to the corresponding total statistics.